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Domain wall spacetimes: Instability of cosmological event and Cauchy horizons

Anzhong Wang*
Department of Astrophysics, Observatorio Nacional, Rua General José Cristino 77, 20921-400 Rio de Janeiro, Rio de Janeiro, Brazil

Patricio S. Letelier†
Department of Applied Mathematics-IMECC, Universidade Estadual de Campinas, 13081-970 Campinas, São Paulo, Brazil
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The stability of cosmological event and Cauchy horizons of spacetimes associated with plane symmetric domain walls is studied. It is found that both horizons are not stable against perturbations of null fluids and massless scalar fields; they are turned into curvature singularities. These singularities are lightlike and strong in the sense that both the tidal forces and distortions acting on test particles become unbounded when these singularities are approached.

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I. INTRODUCTION

It is the general belief [1], often referred to as the no-hair conjecture, that the external gravitational field of a very massive collapsing body will finally relax to a black hole field, described by the three-parameter, (mass, charge, and angular momentum) class of the Kerr-Newman (KN) solutions [2], although some counterexamples exist [3]. Regarding the latter, one of the questions is how much those examples represent the evolution of a realistic collapsing body.

Assuming that the no-hair conjecture is true, we still are left with some problems concerning the internal structure of black holes. The KN black hole possesses a Cauchy (inner) horizon, beyond which the predictability of physics, similar to the case of naked singularities [3], becomes impossible even at the classical level. However, as first noticed by Penrose [4], the Cauchy horizon (CH) is a surface of infinite blueshift, and thus when perturbed by some radiative tails (these tails are always expected to exist [5]) it will be turned into a spacetime singularity. This observation has been verified both by perturbations [6] and by analytic investigations [7, 8]. In particular, Poisson and Israel [8] found that when two oppositely moving null fluids are present, the CH in the Reissner-Nordström (RN) solution is replaced by a curvature singularity, and that the mass parameter on this surface becomes unbounded — this is the so-called mass inflation phenomenon. In view of this enormous mass, the charge and angular momentum become irrelevant, and then the internal is accurately described by the Schwarzschild solution. Thus, the CH actually services as the ultimate everything-proof dam [9], at which the evolution of the spacetime is forced to stop. As a result, the predictability is preserved. For the generic cases, the nature of this singularity, lightlike (null) or spacelike, now is still unclear [10], although according to the strong cosmic censorship conjecture [11] the spacelike nature is more favorable [12].

Motivated partly by the recent studies of the inflationary Universe [13], models including the cosmological constant have been considered [14, 15]. In particular, it is found that, contrary to the KN black hole, the ones with the cosmological constant have a CH which is stable for certain choices of the free parameters. Thus, the problem of the predictability rises again. However, it should be noted that whenever the cosmological constant is different from zero, a cosmological event horizon (CEH) is present [16].

The studies of the internal structure of black holes carried out so far are mainly restricted to the spacetimes with spherical symmetry [10], although some attempts to study spacetimes with axial symmetry have already been initiated [17]. However, because of the complexity of the problems involved, such studies (even in the spherically symmetric case) are frequently frustrating [10]. Therefore, it would be of interest to investigate the above-mentioned problems in the spacetimes which are simpler but in which some of those nontrivial properties of black holes remain. Activity in this direction has already been taken in low-dimensional spacetimes [18].

In this paper, we shall study the stability of the CEH and CH in the usual (3+1)-dimensional spacetimes with plane symmetry, due to the recent discovery of the nontrivial topology of plane domain wall spacetimes [19–21]. In these solutions, CEH's, CH's, and event horizons (EH's) are all present. Since the spacetimes with plane symmetry are easier to handle, they provide a base on which the above issues can be studied in some detail. It might be argued that spacetimes with plane symmetry are not realistic, and that they involve infinitely large masses. In addition, domain walls violate the strong energy condition [22] (but not the weak and dominant ones). However, as we shall see below, they do shed some light on the black hole paradigm.

*Electronic address: wang@on.br
†Electronic address: letelier@ime.unicamp.br

The rest of the paper is organized as follows. In Sec. II some properties of the spacetimes with plane symmetry are briefly reviewed. In Sec. III the instability of the CEH's appearing in a domain wall spacetime [19] is studied. Following a similar line, the instability of the CH's of a supersymmetric plane domain wall [20] is investigated in Sec. IV. Finally, in Sec. V our main conclusions are presented.

In this paper, the units are chosen such that $8\pi G = 1 = c$, where G denotes the gravitational constant, and c the speed of light. The signature of the metric is $+---$.

II. SPACETIMES WITH PLANE SYMMETRY

To facilitate our discussions, in this section we shall briefly review some properties of spacetimes with plane symmetry.[1] The metric in general can be written as [23]

$$ds^2 = f(dt^2 - dz^2) - e^{-U}(dx^2 + dy^2), \tag{1}$$

where f and U are functions of t and z only, and the range of the coordinates is $-\infty < t, z, x, y < +\infty$. The three Killing vectors that characterize the plane symmetry are ∂_x, ∂_y, and $y\partial_x - x\partial_y$. Introducing two null coordinates u and v via the relations

$$t = a(u) + b(v), \qquad z = a(u) - b(v), \tag{2}$$

where $a(u)$ and $b(v)$ are two arbitrary functions of their indicated arguments, subject to $a'(u)b'(v) \neq 0$, where a prime denotes the ordinary differentiation, Eq. (1) now reads

$$ds^2 = 2e^{-M}dudv - e^{-U}(dx^2 + dy^2), \tag{3}$$

with $e^{-M} \equiv 2a'(u)b'(v)f(t,z)$. The corresponding nonvanishing components of the Ricci tensor can be found, for example, in [26]. Because of the symmetry, the Weyl tensor $C_{\mu\nu\lambda\rho}$ has only one "Coulomb" component, given by [26]

$$\begin{aligned}\Psi_2 &\equiv -\frac{1}{2}C_{\mu\nu\lambda\rho}(l^\mu n^\nu l^\lambda n^\rho - l^\mu n^\nu m^\lambda \bar{m}^\rho) \\ &= \frac{1}{6}e^M(M_{,uv} - U_{,uv}),\end{aligned} \tag{4}$$

where $()_{,x} \equiv \partial()/\partial x$, and l^μ, n^μ, m^μ, and $\bar{m}^\mu$ are four null vectors, defined by

$$\begin{aligned}&l^\mu = e^{M/2}\delta^\mu_v, \qquad n^\mu = e^{M/2}\delta^\mu_u, \\ &m^\mu = \frac{e^{U/2}}{\sqrt{2}}(\delta^\mu_x + i\delta^\mu_y), \quad \bar{m}^\mu = \frac{e^{U/2}}{\sqrt{2}}(\delta^\mu_x - i\delta^\mu_y).\end{aligned} \tag{5}$$

Thus, according to the Petrov classifications [27], the spacetimes described by Eq. (1) or Eq. (3) are either Petrov type D ($\Psi_2 \neq 0$) or Petrov type O ($\Psi_2 = 0$). Note that the spacetimes of black holes usually are Petrov type D [1, 27].

On the other hand, one can show that the two null vectors $\nabla_\mu u$ $(= e^{M/2}l_\mu)$ and $\nabla_\mu v$ $(= e^{M/2}n_\mu)$, where ∇ denotes the covariant derivative, define two null affinely parametrized geodesic congruences [28], and that the quantities

$$\begin{aligned}&Q_l \equiv -g^{\mu\nu}\nabla_\mu\nabla_\nu u = e^M U_{,v}\,, \\ &Q_n \equiv -g^{\mu\nu}\nabla_\mu\nabla_\nu v = e^M U_{,u}\end{aligned} \tag{6}$$

represent the rates of contraction of the null geodesic congruences, defined, respectively, by $\nabla_\mu u$ and $\nabla_\mu v$.

III. INSTABILITY OF COSMOLOGICAL EVENT HORIZONS

In 1983, Vilenkin [29] found a solution to the Einstein field equations, which represents a plane domain wall with zero thickness. The solution is given by

$$f = e^{-k|z|}, \qquad U = k(|z| - t), \tag{7}$$

where k is a positive constant. To justify that the above solution indeed represents a domain wall, Widrow [30] considered the fully coupled Einstein-scalar field equations, and found that when far away from the center of the wall, the metric for the Einstein-scalar field equations is indeed well described by Eq. (7), although when near the center they coincide only with the first order. Since in this paper we are mainly concerned with the behavior of the metric at $|z| = \infty$ (as we shall see below, this limit describes the locations of the domain walls' CEH), the description of the wall by Eq. (7) is sufficient for our present purposes.

One of the interesting features of Vilenkin's solution is that at each of the three spatial directions a CEH exists. The ones in the x and y directions are de Sitter-like, and the extensions beyond them are similar to the four-dimensional counterpart given in [22]. In [29], Vilenkin provided an extension beyond the horizon in the z direction, while lately Gibbons [19], among other things, reconsidered this problem and provided another. In this section, we shall first (Sec. III A) give another extension of Vilenkin's solution along a line similar to that of Ref. [21]. The extension is first made independently in each side of the wall, and then glued together. The explicit expressions for such a gluing are given, which are not expected in the general case [31, 32]. In the same subsection, Vilenkin's extension and the interpretation of Vilenkin's domain wall as a closed hypersurface, a bubble [33], are also considered. In Sec. III B the instability of the CEH's is studied.

A. Maximal extension of Vilenkin's solution

Following Ref. [21] (see also Refs. [20] and [34]), let us first make the coordinate transformation

$$u = \alpha^{-1}e^{-k(t+z)/2}, \;\; v = -\alpha^{-1}e^{k(t-z)/2} (z \geq 0), \tag{8}$$

in the region $z \geq 0$, where $\alpha \equiv k/\sqrt{2}$. Then, in terms of

[1]Here we use the definition of plane symmetry originally given by Taub [23]. Recently, this definition was extended to cover a more general situation [24, 25]. Now, the spacetimes defined by Eq. (1) are said to have planar symmetry.

u and v, the metric takes the form of Eq. (3) with

$$M = 0, \qquad U = -\ln(\alpha^2 v^2) \quad (z \geq 0). \tag{9}$$

From Eq. (8), we find

$$uv = -\alpha^{-2} e^{-kz}, \quad \frac{u}{v} = -e^{-kt} \quad (z \geq 0), \tag{10}$$

which shows explicitly the mappings between the (t, z) and (u, v) planes. In particular, the wall $(z = 0)$ is mapped to the hyperbola $uv = -\alpha^{-2}$, while the hypersurface $z = +\infty$ is mapped to the two axes $u = 0$ and $v = 0$, across which the coordinate t becomes spacelike and z timelike. As Gibbons pointed out [19], these two axes actually are the locations of the CEH's [cf. Fig. 1]. From Fig. 1(a) we can see that there essentially exist two walls, each of them located in one of the two branches of the hyperbola $uv = -\alpha^{-2}$. These two walls move towards each other at the beginning with a constant acceleration, and then recede to infinity and behave like Rindler particles [35]. On the other hand, one can show that the extension given by Eq. (9) is the maximal and analytic extension of Vilenkin's domain wall solution for the region $z \geq 0$. This can be seen, for example, by transforming it into the Minkowski spacetime

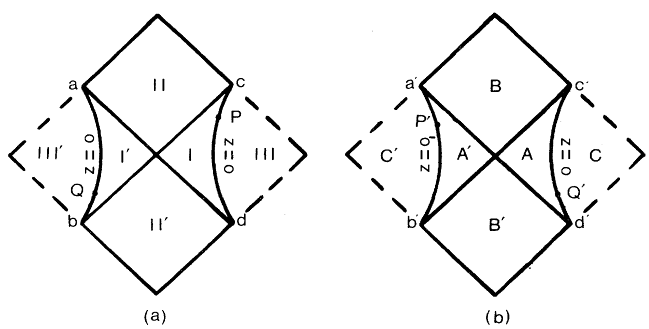

FIG. 1. The Penrose diagram for the extended Vilenkin domain wall spacetime. The spatial coordinates x and y are suppressed. (a) represents the extension in the region where $z \geq 0$. In particular, the region $z \in [0, +\infty)$ is mapped to the regions I and I′, where $\mathrm{I} \equiv \{x^\mu : -\alpha^{-2} \leq uv < 0, u > 0\}$ and $\mathrm{I}' \equiv \{x^\mu : -\alpha^{-2} \leq uv < 0, v > 0\}$ are symmetric with respect to the hypersurface $u = v$ and are causally disconnected. The timelike coordinate t is past directed in region I and future directed in region I′. Regions II ($\equiv \{x^\mu : u, v \geq 0\}$) and II′ ($\equiv \{x^\mu : u, v \leq 0\}$) are two extended regions, while regions III($\equiv \{x^\mu : uv < -\alpha^{-2}, u > 0\}$) and III′ ($\equiv \{x^\mu : uv < -\alpha^{-2}, v > 0\}$) are the regions in the other side of the wall. (b) represents the extension of the spacetime in the region where $z \leq 0$. Because of the reflection symmetry, it can be easily obtained by replacing u, v by $\bar{u}, \bar{v}$, and the regions I, I′, II, II′, III, III′ by A, A', B, B', C, C', respectively. To match the two diamonds together, regions III, III′, C and C' have to be removed. The identifications on the walls are given by Eq. (14). For example, the two points P and Q are identical, respectively, to P' and Q'. The lines $ad, bc, a'd'$, and $b'c'$ are the locations of the cosmological event horizons.

$$\bar{T} = \frac{1}{\sqrt{2}} \left\{ \frac{k^2}{4}(x^2 + y^2)v + (u + v) \right\},$$

$$\bar{Z} = \frac{1}{\sqrt{2}} \left\{ \frac{k^2}{4}(x^2 + y^2)v + (u - v) \right\},$$

$$\bar{X} = \frac{kvx}{\sqrt{2}}, \qquad \bar{Y} = \frac{kvy}{\sqrt{2}}. \tag{11}$$

By using the above equation, Gibbons made the extension for the region where $z \geq 0$. From the above discussions we can see that the only difference between ours and Gibbons' is that in our case we have removed the two regions III and III′ while Gibbons removed only region III′ and kept region III as a part of the extended spacetime. As a result, in our extension, there exist two walls, while in Gibbons' there exists only one wall.

In the region where $z \leq 0$, similarly we make the coordinate transformation

$$\bar{u} = -\alpha^{-1} e^{k(t+z)/2}, \quad \bar{v} = \alpha^{-1} e^{-k(t-z)/2} \quad (z \leq 0). \tag{12}$$

Then, we have

$$\bar{u}\bar{v} = -\alpha^{-2} e^{kz}, \quad \frac{\bar{u}}{\bar{v}} = -e^{kt} \quad (z \leq 0), \tag{13}$$

from which the mappings between the (t, z) and $(\bar{u}, \bar{v})$ planes can be found easily, which is similar to that for the region where $z \geq 0$ [cf. Fig. 1(b)]. In particular, the center of the wall $(z = 0)$ is mapped to the hyperbola $\bar{u}\bar{v} = -\alpha^{-2}$, and the hypersurface $z = -\infty$ to the axes $\bar{u} = 0$ and $\bar{v} = 0$, across which the coordinates t and z exchange their roles.

Assuming Eqs. (8) and (12) to be valid in the neighborhood of the hypersurface $z = 0$, we can immediately find the matching between the two extended regions, which is given by

$$\bar{u} = -(\alpha^2 u)^{-1}, \quad \bar{v} = -(\alpha^2 v)^{-1}. \tag{14}$$

In view of the above equation, we can write the metric in the whole extended spacetime as

$$ds^2 = \begin{cases} 2dudv - \alpha^2 v^2 (dx^2 + dy^2) & (z \geq 0), \\ 2(\alpha^2 uv)^{-2} dudv - (\alpha u)^{-2}(dx^2 + dy^2) & (z \leq 0). \end{cases} \tag{15}$$

From the above expression we can see that the metric coefficients are continuous across the hypersurfaces $uv = -(\alpha u)^{-2}$ but not their first derivatives, which reflects the fact that the walls are located on these surfaces.

It should be noted that instead of gluing the hypersurfaces ab and $a'b'$ together, as indicated in Fig. 1, one can glue each of them with other pieces that are described by Eq. (15). Such a process can repeat infinite times in the transverse direction, so finally we have a spacetime that has a chain structure (cf. Fig. 2). By this way, actually we have infinite number of walls in the whole spacetime and all of them are causally disconnected.

In Ref. [29], Vilenkin gave an extension across the

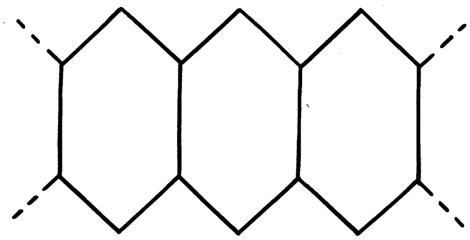

FIG. 2. Instead of identifying the two surfaces ab and $a'b'$ as indicated in Fig. 1, we can glue the two with other pieces that are described by Eq. (15). Repeating this process infinite times in the transverse direction, finally we obtain a spacetime with a chain structure, in which there exists an infinite number of walls. When drawing this diagram, instead of choosing α in Eq. (8) as $\alpha = k/\sqrt{2}$, we have set $\alpha = 1$, so the walls are the vertical lines.

hypersurfaces $|z| = \infty$. Because of the reflection symmetry, it is sufficient to consider the extension in the region where $z \geq 0$, which is performed by introducing two new coordinates T and Z by

$$T = t, \quad Z = \frac{2}{k}(1 - e^{-kz/2}) \quad (z \geq 0). \tag{16}$$

The hypersurface $z = \infty$ is mapped to $Z = 2/k$, and the center of the wall $z = 0$ to $Z = 0$, while the region $z \in [0, +\infty)$ to $Z \in [0, \frac{2}{k})$. From Eqs. (8) and (16), on the other hand, we find

$$u = \alpha^{-1}\left(1 - \frac{k}{2}Z\right)e^{-kT/2},$$

$$v = -\alpha^{-1}\left(1 - \frac{k}{2}Z\right)e^{kT/2} \quad (z \geq 0). \tag{17}$$

The above expressions show that the half of the (T, Z) plane with $Z \geq 0$ is mapped to the three regions I′, I, and III in Fig. 1. Similar to the extension of Gibbons [19], Vilenkin took region III as a part of the extension too. As a result, in Vilenkin's extension, there exists only one wall. However, Vilenkin's extension is different from Gibbons' in that it excludes regions II and II′. Thus, Vilenkin's extension is not the maximal extension.

On the other hand, from Eq. (11) we find that

$$\bar{R}^2 - \bar{T}^2 = -2uv, \tag{18}$$

where $\bar{R}^2 \equiv \bar{X}^2 + \bar{Y}^2 + \bar{Z}^2$. From the above expression, it was concluded that the wall in the Minkowski coordinates (11) is not a plane at all; instead, it becomes a closed hypersurface, a bubble [33]. However, following the considerations given in [21], we argue that the interpretation of the above solution as representing a plane domain wall is more favorable than that as representing a bubble. From Eq. (18) we can see that the coordinate transformations (11) map region I (or I′) in Fig. 1 to the region where $\bar{R} \in [|\bar{T}|, (\bar{T}^2 + 2\alpha^{-2})^{1/2}]$, and the part $D \equiv \{x^\mu : \bar{T}^2 \geq uv \geq 0, u \geq 0\}$, of region II (or $D' \equiv \{x^\mu : \bar{T}^2 \geq uv \geq 0, u \leq 0\}$ of region II′) to the region where $\bar{R} \in [0, |\bar{T}|]$, while the part $E \equiv II - D$ (or $E' \equiv II' - D'$) to a region where the coordinate $\bar{R}$ takes complex values. Therefore, in order to have a geodesically complete spacetime, one is forced to include a region where $\bar{R}$ is complex, which is clearly physically meaningless.

B. Instability of the cosmological event horizons

Now let us turn to consider the stability of the CEH's appearing in the above solution. Because of the reflection symmetry, without loss of generality, in the following we shall focus our attention only in the region where $z > 0$. Then, from Eqs. (6) and (15) we find

$$Q_l = -\frac{2}{v}, \qquad Q_n = 0. \tag{19}$$

Thus, as $v \to 0^-$, we have $Q_l \to +\infty$, which indicates that the CEH at $v = 0$ is not stable against perturbations moving along the null geodesics defined by l^μ. To show that this is indeed the case, it is found sufficient to focus our attention in one of the two diamonds, say, Fig. 1(a). In this region, let us consider the solutions

$$U = -\ln[f(u) + \alpha^2 v^2],$$

$$M = \frac{1}{2}\ln\left(\frac{f(u) + \alpha^2 v^2}{\alpha^2 v^2}\right) - g(u) - h(v), \tag{20}$$

where f, g, and h are arbitrary functions of their indicated arguments. When these functions vanish, the solutions reduce to the one given by Eq. (15) in the region where $z > 0$. When they are different from zero, the corresponding energy-momentum tensor (EMT) in this region is given by

$$T^{\mu\nu} = \rho_1 l^\mu l^\nu + \rho_2 n^\mu n^\nu, \tag{21}$$

where

$$\rho_1 = \frac{(f'g' - f'')e^{-g-h}}{[\alpha^2 v^2 (f + \alpha^2 v^2)]^{1/2}},$$

$$\rho_2 = \frac{2\alpha^2 v h'(v) e^{-g-h}}{[\alpha^2 v^2 (f + \alpha^2 v^2)]^{1/2}}. \tag{22}$$

In the following the arbitrary functions f, g, and h will be chosen such that ρ_1 and ρ_2 are non-negative. Then, we can see that Eq. (21) represents two null dust fluids propagating along the geodesic congruences defined, respectively, by l^μ and n^μ (cf. Fig. 3). In order to consider the fluids as perturbations, we further require that f, g, h, and their derivatives be small.

Note that when $\rho_1 \rho_2 \neq 0$, we can construct two unit vectors u_μ and χ_μ by [28, 36]

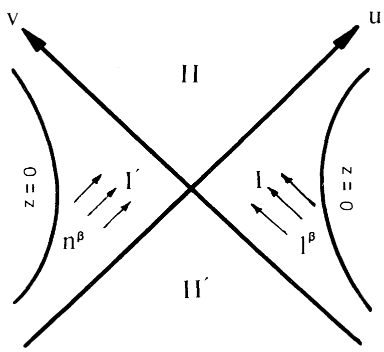

FIG. 3. The projection of the spacetime onto the (u, v) plane. Two null fluids moving toward each other initially in regions I and I′, along the null geodesic congruences defined, respectively, by l^β and n^β. After they collide on the two-surface $u = 0$ and $v = 0$, they form a curvature singularity on the CEH where $v = 0, u \geq 0$. The nature of the singularity is null.

$$u_\mu = \left(\frac{\rho_2}{4\rho_1}\right)^{1/4}\left[n_\mu + \left(\frac{\rho_1}{\rho_2}\right)^{1/2} l_\mu\right],$$

$$\chi_\mu = \left(\frac{\rho_1}{4\rho_2}\right)^{1/4}\left[l_\mu - \left(\frac{\rho_2}{\rho_1}\right)^{1/2} n_\mu\right], \tag{23}$$

such that

$$\rho_1 l^\mu l^\nu + \rho_2 n^\mu n^\nu = \rho(u^\mu u^\nu + \chi^\mu \chi^\nu),$$
$$\rho = (\rho_1\rho_2)^{1/2}, \quad u_\alpha u^\alpha = -\chi_\alpha\chi^\alpha = 1, \quad u_\alpha \chi^\alpha = 0. \tag{24}$$

The above equations show that the sum of two null fluids behaves like an anisotropic fluid: the pressure of it has only one nonvanishing component along the χ^μ direction, and is equal to the energy density of the fluid. Moreover, this anisotropic fluid satisfies all the (weak, dominant, and strong) energy conditions [22].

On the other hand, the combination of Eqs. (4) and (20) yields

$$\Psi_2 = -\frac{\alpha^2 v f'(u) e^{-g-h}}{2[\alpha^2 v^2 (f + \alpha^2 v^2)]^{1/2}}. \tag{25}$$

Thus, because of the existence of the perturbations the spacetime now becomes Petrov type D. In terms of Ψ_2 and $\rho_{1,2}$, the Kretschmann scalar is given by

$$\mathcal{R} \equiv R^{\mu\nu\lambda\delta} R_{\mu\nu\lambda\delta} = 4(6\Psi_2^2 + \rho_1\rho_2). \tag{26}$$

From Eqs. (22) and (25) we find that as $v \to 0^-$, the Kretschmann scalar diverges as v^{-1}. That is, the CEH at the hypersurface $v = 0$ is not stable against the null fluids and are turned into a scalar singularity. The nature of the singularity is null. It should be noted that the divergence of $\mathcal{R}$ is due to the mutual focus of the two null fluids, and that the "Coulomb" gravitational field Ψ_2 remains finite, a phenomenon which was also found in the spherically symmetric case but at the hypersurface of a CH with a nonvanishing cosmological constant (see the second paper quoted in Ref. [15]).

As in the case of the CH's [7, 8], the presence of the null fluid ρ_2 is not essential to the formation of the singularity as indicated by Eq. (19), although it affects the nature of the singularity. This can be seen by the following considerations. Setting $h(v) = 0$, then Eq. (22) gives $\rho_2 = 0$. To see that in the latter case a spacetime singularity is still formed on the CEH, we follow Ref. [7]. We first find a freely falling frame, and then we calculate the Riemann tensor in this frame. Since the components of the Riemann tensor represent the tidal forces experienced by the timelike particles, if any of them becomes unbounded, we conclude that a spacetime singularity is formed. As assumed above, the functions f, g and their derivatives are very small, we see that the timelike geodesics can be well approximated by the ones with $f = g = 0$. For the latter, the timelike geodesics perpendicular to the (x, y) plane are simply given by the tangent vector $\lambda^\mu_{(0)} \equiv dx^\mu/d\tau = E_+\delta^\mu_u + E_-\delta^\mu_v$, where $E_\pm = [E \pm (E^2 - 1)^{1/2}]/\sqrt{2}$, τ denotes the proper time of the test particles, and E the energy. From $\lambda^\mu_{(0)}$ we can construct other three linearly independent spacelike unit vectors $\lambda^\mu_{(a)} (a = 1, 2, 3)$ by

$$\lambda^\mu_{(0)} = E_+\delta^\mu_u + E_-\delta^\mu_v, \quad \lambda^\mu_{(1)} = E_+\delta^\mu_u - E_-\delta^\mu_v,$$
$$\lambda^\mu_{(2)} = e^{U/2}\delta^\mu_x, \qquad \lambda^\mu_{(3)} = e^{U/2}\delta^\mu_y, \tag{27}$$

where U is given by Eq. (20). One can show that such defined four-vectors form a freely falling frame [37]. Computing the Riemann tensor in this frame, we find that one of the nonvanishing components is given by

$$R_{\mu\nu\sigma\delta}\lambda^\mu_{(0)}\lambda^\nu_{(2)}\lambda^\sigma_{(1)}\lambda^\delta_{(2)} = \frac{E_+^2(f'g' - f'')}{2(\alpha v)^2}, \tag{28}$$

which diverges as v^{-2} as $v \to 0^-$. It is interesting to note that the twice integration of the above component with respect to the proper time, which gives the distortion of the test particles, is proportional to $\ln(-v)$ that also diverges as $v \to 0^-$. This is contrary to the case of the CH in the spherically symmetric spacetimes [9, 10]. On the other hand, from Eq. (26) we can see that now the Kretschmann scalar is finite at $v = 0$. As a matter of fact, one can show that when $h(v) = 0$ the other 13 polynomial curvature scalars [38] are also finite at $v = 0$. Thus, the singularity now becomes a nonscalar one [39, 40], but still very strong in the sense that both of the tidal forces and distortion acting on the test particles diverge as the singularity is approached.

In Ref. [41], we have shown that the CEH's are also not stable against a massless scalar field and are turned into scalar singularities. The difference is that there the only nonvanishing component Ψ_2 of the Weyl tensor also diverges on the CEH's.

IV. INSTABILITY OF CAUCHY HORIZONS

In Ref. [20], Cvetič and co-workers studied spacetimes induced from plane supersymmetric domain walls interpolating between Minkowski (M_4) and anti–de Sitter (AdS_4) vacua. It was found that the global structure of the spacetime has a lattice structure quite similar to that of the RN solution but without singularities. The solution is given by Eq. (1) with

$$f = e^{-U} = \begin{cases} 1, & z \to +\infty, \\ (\alpha z)^{-2}, & z \to -\infty, \end{cases} \tag{29}$$

where α is defined as $\alpha \equiv (-\Lambda/3)^{1/2}$, and Λ is the cosmological constant which is negative in the present case. In between these two asymptotic regions, a domain wall is located, and the metric coefficients smoothly interpolate between the two vacuum regions. Since we are mainly concerned with the asymptotic behavior of the spacetime, without loss of generality, we can take the wall as infinitely thin and located on the hypersurface $z = -\alpha^{-1}$ [20]. Then, the spacetime is M_4 for $z > -\alpha^{-1}$ and AdS_4 for $z < -\alpha^{-1}$. On the hypersurface there is a domain wall with its surface energy density given by $\sigma = 2\alpha$. By studying the motion of the test particles, it was found [20] that particles leaving from the wall and moving to the AdS_4 side reach $z = -\infty$ in a finite proper time. Thus, to have a geodesically complete spacetime, one needs to extend the solution beyond $z = -\infty$. After this is done, the spacetime has a lattice structure, and the hypersurface $z = -\infty$ actually represents a CH [20]. For details, we refer the readers to see Ref. [20]. In the following, we shall consider the stability of the CH against null fluids and massless scalar fields.

A. Perturbations of null fluids

Choosing $a(u) = u/\sqrt{2}$ and $b(v) = v/\sqrt{2}$ in Eq. (2), from Eq. (6) we then find that

$$Q_l = -Q_n = -\sqrt{2}\alpha z, \tag{30}$$

in the AdS_4 side. Thus, as $z \to -\infty$, we have $Q_l \to +\infty$ and $Q_n \to -\infty$. Then, we would expect that for the perturbations of a null fluid moving along the null geodesics defined by l^μ, the CH will be turned into spacetime singularity. To illustrate this point, let us consider the solutions

$$M = \begin{cases} \ln[\alpha^2(u-v)^2/2] - g(u), & z \le -\alpha^{-1}, \\ -g(u), & z \ge -\alpha^{-1}, \end{cases} \tag{31}$$

and

$$U = \begin{cases} \ln[\alpha^2(u-v)^2/2], & z \le -\alpha^{-1}, \\ 0, & z \ge -\alpha^{-1}, \end{cases} \tag{32}$$

where $g(u)$ is a smooth function. When it vanishes, the solutions reduce to the domain wall solution of Cvetič *et al.* [20]. When it is different from zero but very small, the solutions represent perturbations on the domain wall background. The corresponding EMT is given by

$$T_{\mu\nu} = \sigma h_{\mu\nu}\delta(z+\alpha^{-1}) + (\rho_1 l_\mu l_\nu + p g_{\mu\nu})[1 - H(z+\alpha^{-1})], \tag{33}$$

where

$$\rho_1 = -\frac{\sqrt{2}g'(u)}{z}e^{-g(u)}, \quad \sigma = 2\alpha e^{-g(u)},$$
$$p = \Lambda(e^{-g(u)} - 1), \quad h_{\mu\nu} = g_{\mu\nu} - \frac{\xi_\mu \xi_\nu}{\xi_\lambda \xi^\lambda},$$
$$\xi_\mu \equiv \frac{e^{M/2}}{\sqrt{2}}(\delta^u_\mu - \delta^v_\mu), \tag{34}$$

$\delta(x)$ denotes the Dirac delta function, and $H(x)$ the Heaviside function, which is one for $x \ge 0$ and zero for $x < 0$. Provided that $\rho_1 \ge 0$, we can see that the solutions given by Eqs. (31) and (32) represent perturbations of a fluid in the AdS_4 region, which is described by the last term in the right-hand side of Eq. (33). This fluid is type II in the sense defined in [22]. In the present case, since $g(u)$ is very small, we have $p \approx 0$. Thus, practically the fluid is null. As before, this particular class of single null fluids cannot form a scalar singularity, but, as we shall show below, it does form a nonscalar one. To see this, we calculate the components of the Riemann tensor in a freely falling frame, which is now given by Eq. (27) but with

$$E_\pm = (E(\alpha z)^2 \pm \{(\alpha z)^2[(E\alpha z)^2 - 1]\}^{1/2})/\sqrt{2}, \tag{35}$$

where E is the energy of the test particles. Then, one can show that one of the nonvanishing components of the Riemann tensor is given by

$$R_{\mu\nu\sigma\delta}\lambda^\mu_{(0)}\lambda^\nu_{(2)}\lambda^\sigma_{(0)}\lambda^\delta_{(2)} = \alpha^2 - \frac{g'(u)E_+^2}{\sqrt{2}z}. \tag{36}$$

On the other hand, one can also show that as $z \to -\infty$ we have $z \approx \tau^{-1}$, where τ is the proper time of the test particles. Thus, from Eqs. (35) and (36) we can see that both the tidal forces and distortions acting on the test particles become infinite as $z \to -\infty$. That is, the CH on $z = -\infty$ is indeed turned into a spacetime singularity, and the nature of this singularity, contrary to the spherically symmetric case [9, 10], is strong, although Ψ_2 is still zero, as one can easily show from Eqs. (4), (31), and (32).

B. Perturbations of massless scalar fields

In order to construct perturbations that turn the CH into a scalar singularity, one way is to consider two oppositely moving null fluids, another is to consider perturbations of a massless scalar field, similar to that of Ref. [41]. It should be noted that the specific form of the perturbations, two null fluids, massless scalar fields, or any of others, is not important to the formation of a scalar singularity. What is really important in our analysis is that the perturbations have to have the two nonzero Ricci scalars Φ_{00} and Φ_{22}, where $\Phi_{00} \equiv (R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R)l^\mu l^\nu$ and $\Phi_{22} \equiv (R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R)n^\mu n^\nu$. They represent the mutual focus between the matter components of the perturbations moving along the two null geodesic congruences defined by l^μ and n^μ [42], and the Kretschmann scalar is proportional to $\Phi_{00}\Phi_{22}$ [38].

The perturbations of a massless scalar field on the above domain wall background can be studied by the following specific solution that is given by Eq. (1) with

$$f = \begin{cases} 1, & z \geq -\alpha^{-1}, \\ (\alpha z)^{-2}, & z \leq -\alpha^{-1}, \end{cases}$$
$$U = -\ln(f) - \ln(t_0 - t). \tag{37}$$

The corresponding EMT is given by

$$T_{\mu\nu} = 2\alpha(t_\mu t_\nu - x_\mu x_\nu - y_\mu y_\nu)\delta(z + \alpha^{-1}) + \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}, \tag{38}$$

where

$$\phi = \frac{1}{\sqrt{2}}\ln(t_0 - t), \quad \phi_{;\mu\nu}\, g^{\mu\nu} = 0, \tag{39}$$

and t_0 is an arbitrary constant. The fact that the particular solution (39) is singular at $t = t_0$ will play no role in our analysis, since we are interested in the limit $|z| \to -\infty$. Equation (38) shows that the solution (37) indeed represents a massless scalar field ϕ on the background of the domain wall of Eq. (29). The corresponding Kretschmann scalar is given by

$$\mathcal{R} \equiv R^{\mu\nu\lambda\delta}R_{\mu\nu\lambda\delta} = \begin{cases} \frac{\alpha^2}{4(t_0-t)^4}[96(t_0 - t)^4 + 8(t_0 - t)^2 z^2 + 3z^4], & z \leq -\alpha^{-1}, \\ \frac{3}{4(t_0-t)^4}, & z \geq -\alpha^{-1}. \end{cases} \tag{40}$$

Clearly, as $z \to -\infty, \mathcal{R}$ diverges like z^4. Thus, because of the presence of the massless scalar field, the CH is turned into a spacetime singularity. By considering the components of the Riemann tensor in a freely falling frame, one can show that this singularity is strong. In fact, we find that one of them is given by

$$R_{\mu\nu\sigma\delta}\lambda^\mu_{(0)}\lambda^\nu_{(2)}\lambda^\sigma_{(0)}\lambda^\delta_{(2)} = \alpha^2\left[1 + \frac{\alpha^2 z^4}{4(t_0 - t)^4}\right], \tag{41}$$

which diverges like z^4 as $z \to -\infty$. On the other hand, we find that as $z \to -\infty$ we have $z \approx \tau^{-1}$, where τ, as before, is the proper time of the test particles.

Inserting Eq. (37) into Eq. (4), we have

$$\Psi_2 = \begin{cases} -(\alpha z)^2\,[12(t_0 - t)]^{-1}, & z \leq -\alpha^{-1}, \\ [12(t_0 - t)]^{-1}, & z \geq -\alpha^{-1}. \end{cases} \tag{42}$$

Thus, as $z \to -\infty$, Ψ_2 diverges like $z^2 \approx \tau^{-2}$. Contrary to the perturbations of a null fluid, now we have a "mass inflation phenomenon" (recall that in the spherically symmetric case, Ψ_2 is proportional to the mass parameter). Even though the analysis was carried out with a very particular solution, because of the arguments presented at the beginning of this subsection, we believe that the conclusions are valid for a large class of scalar field perturbations.

In addition to the null singularity occurring on $z = -\infty$, there is also a spacelike singularity on $t = t_0$. That is, initially the CH is turned into a null curvature singularity. However, as the time is developing, the spacetime collapses. At the moment $t = t_0$, the spacetime collapses into a spacelike singularity, and the null one is finally replaced by the spacelike one [10, 43]. This fact depends on the particular form of (39). We also believe that solutions of the field equations presenting a similar singular behavior will produce spacetimes with similar singular structure.

V. CONCLUSIONS

In this paper, we have shown that the CH's appearing in the plane domain wall solution of Cvetič *et al.* [20] are not stable against both a null fluid and a massless scalar field, and are turned into strong curvature singularities. In the perturbations of a null fluid, the divergence of the tidal forces and distortions of the test particles is purely due to the null fluid, and the Weyl tensor vanishes identically. However, in the perturbations of a massless scalar field, it is due to both the scalar field and the "Coulomb" gravitational field Ψ_2. Therefore, a phenomenon similar to mass inflation occurs in the latter case but not in the former.

On the other hand, we have also shown that the CEH's appearing in Vilenkin's plane domain wall spacetime are not stable against null fluids and massless scalar fields. They are all turned into strong spacetime singularities, as both the tidal forces and the distortions of the test particles diverge as these singularities are approached. Regarding this result, a natural question is: Is the CEH appearing in the KN–de Sitter solutions also unstable? To have a definite answer, one way is to consider the perturbations along a line given in Refs. [7, 8]. Work in this direction will be reported somewhere else.

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